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Enhanced network learning model with intelligent operator for the motion reliability evaluation of flexible mechanism

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ABSTRACT

The evaluation of flexible mechanism involving multi-body dynamics with high nonlinearity and transients urgently requires an efficient evaluation method to enhance its reliability and safety. In this work, an enhanced network learning method (ENLM) is proposed to improve the modeling precision and simulation efficiency in flexible mechanism reliability evaluation, by introducing generalized regression neural network (GRNN) and multi-population genetic algorithm (MPGA) into extremum response surface method (ERSM). In the ENLM modeling, the ERSM is adopted to reasonably handle transients (time-varying) problem in motion reliability analysis by considering one extreme value in whole response process; the GRNN is applied to address high-nonlinearity in surrogate modeling; the MPGA is utilized to find the optimal model parameters in ENLM modeling. In respect of the developed ENLM, the motion reliability of two-link flexible robot manipulator (TFRM) was evaluated, with regard to the related input random parameters to material density, elastic modulus, section sizes, and deformations of components. In term of this study, it is illustrated that (i) the comprehensive reliability of flexible robot manipulator is 0.951 when the allowable deformation is 1.8×10^{-2} m; (ii) the maximum deformations of member-1 and member-2 obey normal distributions with the means of 1.45×10^{-2} m and 1.69×10^{-2} m as well as the standard variances of 6.77×10^{-4} m and 4.08×10^{-4} m, respectively. The comparison of methods demonstrates that the ENLM improves the modeling precision by 3.29% and reduces the simulation efficiency by 1.19 s under 10 000 simulations, and the strengths of the ENLM with high modeling precision and high simulation efficiency become more obvious with the increase of simulations. The efforts of this study provide a learning-based reliability analysis way (i.e., ENLM) for the motion reliability design optimization of flexible mechanism and enrich mechanical reliability theory.

1. Introduction

Flexible mechanism is one of important parts in mechanical system in spacecraft, aircraft, robotics, and so forth [1]. The use of flexible mechanism is to transmit force loads and change the movement, so that it severely influences the safety and robustness of mechanical system [2]. With the advances of aerospace and robot technologies, the investigation on flexible mechanism attracts a lot of attention with the emphasis on the control strategy [2] and the modeling and solution of dynamic equation [3–6]. Therefore, it is necessary to evaluate the reliability of flexible mechanism to ensure the stable operation of mechanical system.

Numerous investigations on reliability analysis in many fields lead to the rapid development of reliability analysis approaches [7–10]. Reliability methods may be decomposed into direction method and surrogate model method. The direction method is based on the simulation and analysis on the true model of study objects based on Monte Carlo (MC) method. Usually, the true model is complex finite model (FE), and endures complex boundary conditions and multi-physical loads, so that the computation of FE-based reliability analysis is too large-scale to be implemented in general computing platform. As an alternative approach, surrogate model method (also called response surface method, RSM), an important reliability analysis approach, is being widely applied and rapidly developed, which can largely improve the computational efficiency of reliability analysis relative to the direct simulation method. Fei et al. proposed decomposed-coordinated surrogate modeling strategy for compound function approximation and a turbine blisk reliability evaluation [11]. Zhang et al. studied

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the fuzzy multi-SVR (support vector machine of regression) learning method for the reliability-based design optimization of turbine blades [12]. Lu et al. developed improved Kriging with extremum response surface method for structural dynamic reliability and sensitivity analyses [13]. Li et al. employed support vector machine in structural reliability analysis [14]; Jiang et al. studied active leaning Kriging method for time-dependent reliability analysis [15]; Xiong et al. presented a double weighted stochastic RSM for reliability analysis [16]; Gavin et al. gave the RSM-based high-order limit state functions for reliability analysis [17]; Bai et al. gave a response surface-based structural reliability analysis method with the consideration of non-probability convex model [18]; Dai et al. applied artificial neural network (ANN) with high accuracy and nonlinear mapping capability, to the regression analysis of complex structure limit state function to improve the computational accuracy of reliability [19]. Liao et al. discussed multiaxial fatigue life prediction framework of compressor discs considering notch effects [20]. Although the advanced surrogate model approaches hold acceptable accuracy, uses of them only focus on the reliability analysis of structures rather than flexible mechanism.

Compared to the studied structures, the main function of mechanisms is to transmit and change the movement of components in mechanism system with time, by the forms of displacement and deformation [2,3]. Due to the complexity of relative motion, the components and mechanism system are so flexible that the design and analysis of flexible mechanism with kinetic equations involve high order differential equation set [21]. Obviously, the flexible mechanism analysis involves nonlinear calculation, time dependence (transients) and interaction among many components. In this case, it is inevitable that the reliability analysis of flexible mechanism involves large-scale calculation and strong coupling. When the surrogate model methods effectively used in structural reliability analysis are directly applied to the reliability evaluation of flexible mechanisms, it is difficult to acquire acceptable analytical efficiency and accuracy. To overcome the issues, Song et al. developed dynamic neural network method based on extremum response surface method to improve the probabilistic analysis of flexible mechanism in analytical precision [22]; Gao et al. adopted an adaptive neural network to control the vibration of a flexible aircraft wing system [23]; Zhang et al. investigated the reliability evaluation and topology optimization of compliant mechanism by using level set method and the first order reliability method [24]; Ouyang et al. studied the pole assignment for control of flexible link mechanisms using FE model and surrogate modeling approach [25]. To improve the computational efficiency, Zhang et al. developed extremum RSM (ERSM)-based quadratic polynomials for the reliability analysis of flexible manipulator by combining flexible multi-body dynamics with modal comprehensive method and modal truncation technique [26]. And the ERSM was introduced into machine learning approaches (such as ANN, SVM, Kriging model, and so forth) to enhance the probabilistic design and optimization of complex structures and revealed the ability of the method in handling the transients [27–31]. Yang et al. proposed back propagation-ANN (BP-ANN) for the strength reliability analysis of flexible mechanism [32,33]. These works verified the ERSM and BP-ANN model which can improve the simulation efficiency and accuracy in flexible mechanism reliability analysis to some extent. However, these methods cannot still satisfy the motion reliability analysis of flexible mechanism in engineering. This is determined by the three aspects, i.e., (i) flexible mechanism analysis involves time-varying (transient) characteristics. In the stage of extracting samples, there is not an effective way to reasonably handle the transient features and seriously influences both simulation efficiency and surrogate modeling accuracy; (ii) in fact, the reliability analysis of flexible mechanism has higher nonlinear feature, which cannot be perfectly addressed by these methods, because these models hardly approximate to high-nonlinear flexible mechanism analysis; (iii) in surrogate modeling, the used training algorithms is based on low dimensional polynomials and usually involves the problem of local optimization and nonconvergence in model parameter optimization process. Therefore, it is hard to ensure the model parameters (i.e., weights and thresholds) to be optimal that reduce the effectiveness of the established surrogate model.

To conquer the transients of dynamic structural reliability analysis, the ERSM was developed and verified to be resultful [26,29], because the ERSM only considers the global extreme value of output response in each simulation process rather than its local extremum values, and is promising to ensure the precision of samples extracted and improve the efficiency of simulation by only focusing on the extreme value. Therefore, ERSM is adopted to address the transients of flexible mechanism analysis in sample extraction with the simulation of kinetic equations. With the rapid development of neural network technology, generalized regression neural network (GRNN) was proposed [34]. The GRNN skillfully address the nonlinear problem by a strong nonlinear mapping from high-dimensional space to low dimensional space. Meanwhile, the GRNN has been demonstrated to be high computational precision and efficiency in structural design optimization [35–39]. The emergence of the GRNN offers the useful insight for the solution of high nonlinearity in flexible mechanism reliability analysis. In seeking for the optimal model parameters, genetic algorithm (GA) was often employed in structural design in the past [13,39]. However, the GA holds the premature problem in parameters optimization [40], due to the dismatch or discordant of many processes such as fitness value, crossover and mutation probabilities, population size, termination criterion, and so forth. Recently, multi-population GA (MPGA) was proposed to search the optimal values of model parameters [41], and was validated that the MPGA holds flexible and adaptive design space exploration and avoid the influence of the plateau-like function profile of MLE. Besides, the MPGA uses multiple populations with different control parameters for optimization iterations, and thus breaks the premature problem of single population evolution for the traditional GA [39,40]. Substantially, the MPGA originates from GA and inherits natural selection and genetic characteristics. The use of MPGA in flexible mechanism reliability analysis is the solution of the third issue.

The objective of this paper is to propose an enhanced network learning method (ENLM) by integrating the strengths of ERSM, GRNN and MPGA, to improve the modeling accuracy and simulation efficiency of flexible mechanism motion reliability analysis caused by transients, high-nonlinearity and hyperparameters. Herein, the ERSM is applied to reasonably handle the transients (time-varying) of motion reliability analysis by considering one extreme value rather than the whole response process. The GRNN is employed to address the high-nonlinearity in the surrogate modeling of flexible mechanism; the MPGA is adopted to find the optimal model parameters in the ENLM modeling. The strengths of the three methods collectively guarantee the modeling accuracy and simulation efficiency in flexible mechanism motion reliability analysis. Based on the proposed ENLM, the reliability analysis of two-link flexible mechanism (TFRM) was implemented with regard to random variables and mechanism deformation. In respect of the comparison of methods with MC method, ERSM and GRNN, the presented ENLM is validated with high modeling precision and good simulation efficiency.

The remainder of this paper is organized as follows. Section 2 discusses the developed ENLM, involving ERSM, ENLM and MPGA. The basic thought of flexible mechanism motion reliability analysis with the ENLM and MPGA is constructed in Section 3. The motion reliability analysis of TFRM is applied to validate the modeling precision and simulation efficiency of the proposed ENLM in Section 4. Section 5 gives the conclusions summarized in this study.

2. Enhanced network learning method

2.1. Basic thought of extremum response surface method

In dynamic reliability analysis, the output response is varying with time and work. The traditional approach of handling the transient response is to establish multiple analytical models at different time points in a time domain [0, T]. It is undoubtedly that this way largely increases the complexity and time cost in simulations. How to effectively process the transients is one key issue in dynamic reliability analysis with high computational efficiency. To address this issue, the ERSM was developed [13]. The ERSM only considers one extremum (maximum value or minimal value) of output response process instead of the whole process in a time domain, so that the ERSM can reduce computational burden in dynamic reliability analysis, relative to the traditional method [13,26]. In this paper, the ERSM is adopted to handle the transient problem of the motion reliability evaluation of flexible mechanism.

Assuming $\boldsymbol{X}$ and y_e are input variables set (a vector) and an extremum output response, the ERSM model $y_e(\boldsymbol{X})$ can be expressed by

$$y_e\left(\mathbf{X}\right)=\left\{y_e^{(j)}\mathbf{X}^{(j)}\right\}_{j=1}^{m} \quad (1)$$

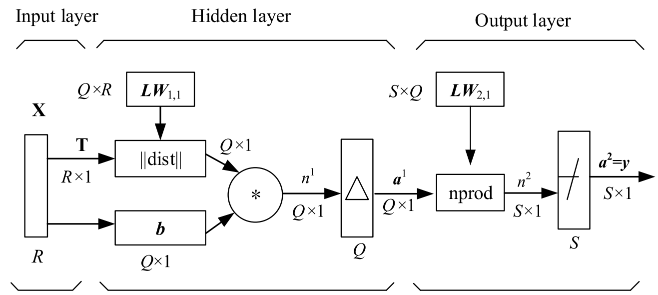
in which m is the total number of samples; $\boldsymbol{X}^{(j)}$ stands for the jth set of input samples; $y_e^{(j)}$ indicates the extreme value of output response in time domain which is correlated with the jth set of input samples $\boldsymbol{X}^{(j)}$.

In Eq. (1), we can find that the ERSM is only a thought or strategy. The ERSM model has been derived on basis of quadric polynomials in the previous works [26,42]. These models were usually inefficient and inaccurate because it is difficult for quadric polynomials and least square method to accurately reflect and describe the high-nonlinearity between numerous input parameters and output response in flexible mechanism analysis. In this case, it is necessary to develop an efficient model as the basis model of the ERSM to make up with the weakness of quadric polynomials and least square method in high-nonlinear notion reliability evaluation of flexible mechanism.

2.2. Enhanced network learning method

Generalized regression neural network (GRNN) has strong nonlinear mapping capacity from low dimensional space into high-dimensional space [34]. By the mapping relationship, the high-nonlinear relationship problem can be resolved by solving the low-nonlinear relationship problem in high-dimensional space. Therefore, it is promising for GRNN to skillfully address the high-nonlinear problem in the motion reliability analysis of flexible mechanism. However, the transients of the motion reliability analysis of flexible mechanism cannot be processed yet, by the GRNN. To address this issue, enhance network learning method (ENLM) is developed by the GRNN as the basis model of ERSM, which absorbing both the nonlinear processing abilities of the GRNN and the transient processing ability of the ERSM. Obviously, the ENLM is promising to improve analytical precision and efficiency in the motion reliability analysis of flexible mechanism. In light of the features of GRNN, the ENLM is a feedforward network model with the theory of nonlinear regression, which comprises input layer, hidden layer and output layer. The schematic diagram of ENLM model is shown in Fig. 1.

In input layer, the input and output matrixes correlated with training samples, $\boldsymbol{X}$ and $\boldsymbol{T}$, are



Note: $\boldsymbol{X}$—input samples matrix; $\boldsymbol{T}$—output samples matrix; $Q\times R$—the dimensions of matrix $\boldsymbol{LW}_{1,1}$ which is the weighted matrix in hide layer, where Q and R are the number of training samples and input parameters, respectively; ||dist||—Euclidean distance function; $\boldsymbol{b}$—the threshold of Q neural cells in hidden layer; n^1—the network vector of hide layer; △—transfer (Gauss) function; $\boldsymbol{a}^1$—the output of neuro cell; $S\times Q$—the dimensions of matrix $\boldsymbol{LW}_{2,1}$ which is connection threshold value between hide layer and output layer, where S is the number of output parameters; nprod—the weight function of output layer; n^2—the network vector of output layer; ⁄—the purelin transfer function of output layer; $\boldsymbol{y}=a^2$—the outputs of neuro cell in output layer.

Fig. 1. Schematic diagram of enhanced network learning method model.

$$\boldsymbol{X}=\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1Q} \\ x_{21} & x_{22} & \cdots & x_{2Q} \\ \cdots & \cdots & \cdots & \cdots \\ x_{R1} & x_{R2} & \cdots & x_{RQ} \end{bmatrix} \qquad \boldsymbol{T}=\begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1Q} \\ t_{21} & t_{22} & \cdots & t_{2Q} \\ \cdots & \cdots & \cdots & \cdots \\ t_{S1} & t_{S2} & \cdots & t_{SQ} \end{bmatrix} \quad (2)$$

in which x_{ji} ($j=1,2,\ldots,R$; $i=1,2,\ldots,Q$) indicates the sample value of jth input parameter in ith training samples; t_{ji} denotes jth output sample correlated with ith training samples.

In hidden layer, the number of neurons was equal to the number of training samples. Weight function can be expressed by Euclidean distance function. Thus, the weight matrix in implicit layer is demoted by

$$\boldsymbol{LW}_{1,1}=\boldsymbol{X}^T \quad (3)$$

The thresholds $\boldsymbol{b}$ of Q neural cells in hidden layer are

$$\boldsymbol{b}=\left[b_1,b_2,\cdots b_Q\right]^T \quad (4)$$

in which $b_1=b_2=\cdots=b_Q=0.8326/\sigma$ where σ illustrates the smooth factor of Gauss function.

Gaussian radial basis function is usually adopted as the transfer function of hidden layer. In hidden layer, the number of neurons Q and training samples is the same, and one training sample has a neuron. Based on the values of weight matrix and thresholds, the output a_i^j of ith hidden layer neuron is

$$a_i^j=\exp\left(-\frac{0.8326\left\|\boldsymbol{LW}_{1,i}-\boldsymbol{x}_j\right\|^2}{\sigma}\right) \quad (5)$$

in which $\boldsymbol{LW}_{1,i}$=[x_{h1}, x_{h2}, ..., x_{hR}]^T subject to $h=1,2,\ldots,Q$ indicates the vector of ith implicit layer in $\boldsymbol{LW}_{1,1}$; $\boldsymbol{x}_j$=[x_{j1}, x_{j2}, ..., x_{jR}]^T is jth training samples vector. $a^j=\left[a_1^j,\cdots,a_i^j,\cdots a_Q^j\right]$ is Q nerve cells vector for jth input samples.

Regarding the $\boldsymbol{LW}_{2,1}$ as the output matrix of training samples set, i.e.,

$$\boldsymbol{LW}_{2,1}=\boldsymbol{T} \quad (6)$$

and the third layer of GRNN as the output layer, in respect of (5) and (6), the vector n^j is

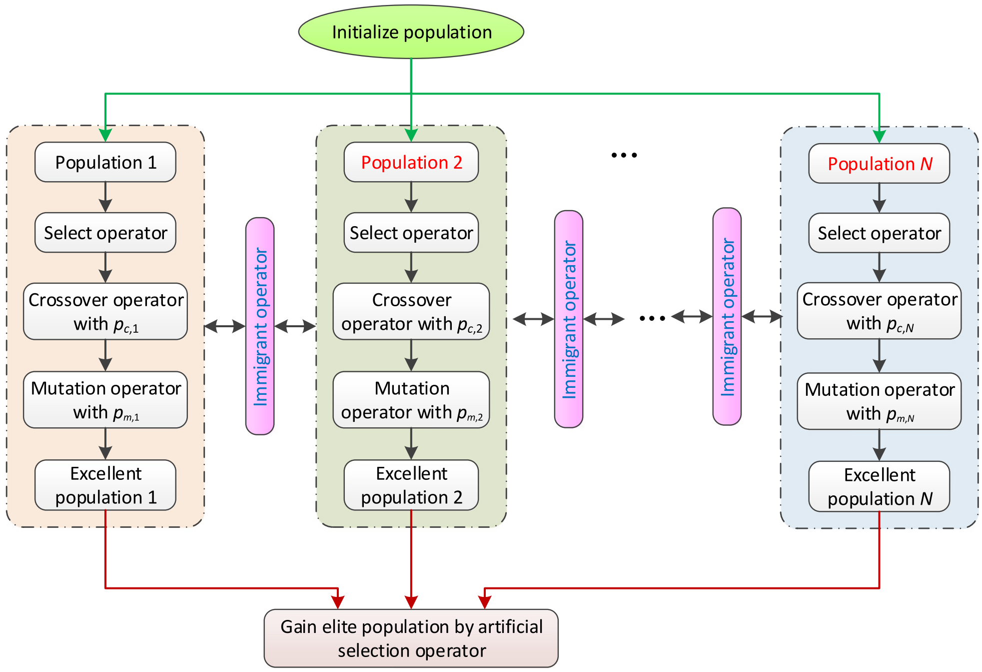


Fig. 2. The structure chart of multi-population genetic algorithm (MPGA).

$$n^j = \frac{\boldsymbol{LW}_{2,1}\left[\boldsymbol{a}^j\right]^T}{\sum_{i=1}^{Q} a_i^j} \tag{7}$$

Regarding the line transfer function y^j=purelin(n^j), the GRNN model subject to the response of jth training samples is

$$y^j = \text{purelin}\left(n^j\right) = \frac{\boldsymbol{LW}_{2,1}\left[\boldsymbol{a}^j\right]^T}{\sum_{i=1}^{Q} a_i^j} \tag{8}$$

With regard to the extremum thought of ERSM in Eq. (1), the model of ENLM can be established as

$$y^j_{\max} = \max\left\{\frac{\boldsymbol{LW}_{2,1}\left[\boldsymbol{a}^j\right]^T}{\sum_{i=1}^{Q} a_i^j}\right\} \tag{9}$$

As shown in the above analysis, the weights ($\boldsymbol{LW}_{1,2}$ and $\boldsymbol{LW}_{2,1}$) and thresholds ($\boldsymbol{b}$) directly determine the effectiveness and feasibility of the established ENLM model in Eq. (9). These hyperparameters in ENLM model are denoted by $\boldsymbol{\theta}$=[$\boldsymbol{LW}_{1,2}$, $\boldsymbol{LW}_{2,1}$, $\boldsymbol{b}$]. Thus, the modeling process of the ENLM can be actually transformed into searching for the optimized values of the hyperparameters $\boldsymbol{\theta}$=[$\boldsymbol{LW}_{1,2}$, $\boldsymbol{LW}_{2,1}$, $\boldsymbol{b}$]. In the traditional approach on the optimization of the hyperparameters is least square method [43]. When the least square approach is applied to find the hyperparameters $\boldsymbol{\theta}$ of ENLM model, numerous iterations are required for the large-scale parameters and high-nonlinear problems, and the results also easy immerses in local optimum. Therefore, the find of the optimal parameters based on the least square approach exists local optimum problem and costs too much time. It is urgent to find an efficient technique to optimize the hyperparameters $\boldsymbol{\theta}$=[$\boldsymbol{LW}_{1,2}$, $\boldsymbol{LW}_{2,1}$, $\boldsymbol{b}$] as ENLM modeling.

2.3. Intelligent operator design of MPGA

The hyperparameters $\boldsymbol{\theta}$ is the key factor of affecting the modeling accuracy of ENLM. To overcome the weakness of the least square method in search for the model hyperparameters $\boldsymbol{\theta}$, genetic algorithm (GA) was applied to find the optimal values of hyperparameter $\boldsymbol{\theta}$, and was demonstrated to hold strong robustness and global search ability [39,40]. However, we find that the GA exists premature problem due to fitness value, crossover and mutation probabilities, population size, termination criterion and so forth [41]. To address the above issue, we attempt to adopt the MPGA to search the optimal values of hyperparameters $\boldsymbol{\theta}$. Relative to the GA, the MPGA as an intelligent operator holds flexible and adaptive design space exploration, and avoid the influence of the plateau-like function profile of least square method. Besides, the MPGA uses multiple populations with different control parameters for optimization iterations which can breaks the limitation of single population evolution of GA in premature problem [42]. Substantially, the MPGA originates from GA and inherits natural selection and genetic characteristics. The optimal solution of objective function can be gained via successive iterations with selection, crossover and mutation. The structure chart of MPGA is shown in Fig. 2.

As shown in Fig. 2, we first create N initial populations and gain excellent populations with multi-population coevolution by selection operator with roulette method, crossover operator with $p_{c,l}$, and mutation with $p_{m,l}$, in which $p_{c,l}$ and $p_{m,l}$ ($l = 1, 2, \ldots, N$) are the crossover and mutation probabilities of the lth population. Then, we obtain elite population which is structured with the optimal individual in each excellent population selected by artificial selection operator. Therein, the MPGA gives consideration to global search and local search, by different control parameters (crossover probability $p_{c,l}$ and mutation probability $p_{m,l}$). Immigrant operators play an important role in this algorithm, which establish a bridge for information exchange among multiple populations. The elite population cannot execute the operators of selection, crossover and mutation, to avoid the destruction and loss of the optimal individuals in the evolution process, and is also the basis of the optimal termination. Besides, we select the minimum preserving generations for the optimal individual as the terminal criterion in this paper. Moreover, an optimal problem is resolved by minimizing an objective function typically. Accordingly, when an optimization problem with a maximization objective function is given, we need to convert the function into a minimization of the corresponding

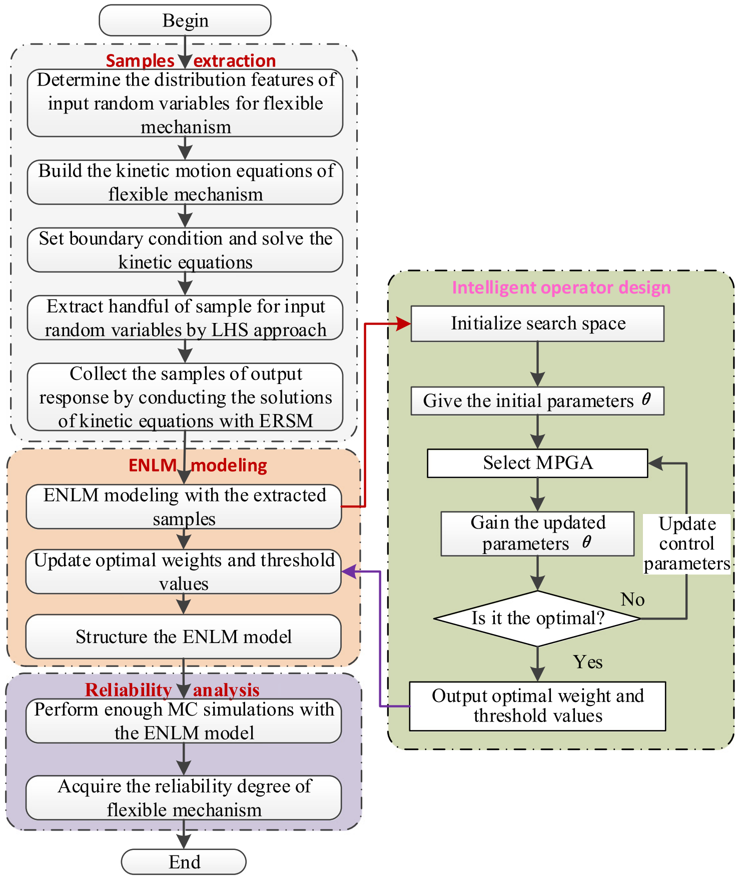


Fig. 3. Motion reliability analysis procedure of flexible mechanism with ENLM and MPGA.

function, namely the negative value of the objective function, to implement the optimization.

Based on the above principle, obviously, the MPGA as an intelligent operator does not only hold flexible and adaptive design space exploration and avoid the influence of the plateau-like function profile of least square method, but also can break the limitation of single population evolution of GA in premature problem by adopting multiple populations with different control parameters for optimization iterations. Therefore, the MPGA has the potential to find the optimal parameters $\boldsymbol{\theta}$ in the ENLM modeling, so that the ENLM is accurately modeled for the reliability analysis of flexible mechanism in this paper.

3. Reliability analysis method with the ENLM for flexible mechanism

When y^* indicates the allowable values of flexible mechanism motion (deformation or displacement), in respect of Eq. (9) the limit state function of flexible mechanism motion can be expressed by

$$Z = y^* - y^i_{\max} \tag{10}$$

here $Z > 0$ indicates that the mechanism is safe, and vice versa.

When the random input variables impacting on the motion are mutually independent and obey normal distributions, the output response of the flexible mechanism analysis distributes normally [28]. Hence the reliability degree P_r can be computed by

$$P_r = \Phi\left(\frac{\mu_Z}{\sqrt{D_Z}}\right) \approx \frac{N_r}{N_{\text{total}}} \tag{11}$$

where μ_Z and D_Z are the mean and variance of the limit state function Z of flexible mechanism motion, respectively; N_r and N_{total} indicate the number of samples in safety domain and the number of total samples tested, respectively.

In term of the proposed ENLM with MPGA, the procedure of the motion reliability analysis of flexible mechanism is drawn in Fig. 3.

As illustrated in Fig. 3, the procedure of the motion reliability evaluation of flexible mechanism with ENLM and MPGA involves four parts, i.e., samples extraction, ENLM modeling, intelligent operator design and reliability analysis. The samples extraction is to collect enough and handful of samples by LHS technique to train the ENLM model. The objective of the ENLM modeling to gain efficient ENLM models by applying the MPGA to find the optimal hyperparameters θ, for the reliability evaluation of flexible mechanism. The intelligent operator design is to search the optimal hyperparameters $\boldsymbol{\theta}$ in ENLM model, comprising the connection weights ($\boldsymbol{LW}_{1,2}$ and $\boldsymbol{LW}_{2,1}$) and threshold values $\boldsymbol{b}$ (or optimal smooth factors σ) among input layer, hidden layer and output layer. The aim of reliability analysis is to assess the performance and reliability of flexible mechanism under operation and obtain its motion reliability degree. The basic process of the motion reliability analysis of flexible mechanism with the developed ENLM and MPGA is summarized as follows:

***Step* 1**: Build the kinetic motion equations of flexible mechanism in light of multi-body dynamics principle [3].

***Step* 2**: Set boundary conditions in view of the numerical features of random inputs, perform the deterministic analysis based on the kinetic motion equations, and then determine the design point with the minimum displacement/deformation with regard to the extremum thought of ERSM, for the motion reliability estimation of flexible mechanism.

***Step* 3**: Obtain a handful of input samples by adopting Latin hypercube sampling (LHS) method [44] and the extreme values of flexible mechanism motion, to structure the set of samples including the set of training samples and the set of testing samples.

***Step* 4**: Train the ENLM model based on the acquired samples, by using the MPGA method [45] to determine the hyperparameters $\boldsymbol{\theta}$ of ENLM model comprising the connection weights ($\boldsymbol{LW}_{1,2}$ and $\boldsymbol{LW}_{2,1}$) and threshold values $\boldsymbol{b}$(or optimal smooth factors σ).

***Step* 5**: Structure the limit state function of flexible mechanism reliability analysis based on the built ENLM model, referencing the allowable deformation of flexible mechanism.

***Step* 6**: Verify the precision of ENLM model by the testing samples by comparing with the true value computed by the kinetic (motion) equation of flexible mechanism. If satisfied, implement ***Step* 7**; conversely, return to ***Step* 4**.

***Step* 7**: Perform the motion reliability analysis of flexible mechanism by extracting enough simulations with the MC method to determine reliability degree in line with the Eq. (11).

4. The motion reliability analysis of flexible mechanism

4.1. Problem description

As one of typical flexible mechanisms, two-link flexible robot manipulator (TFRM, short for) is selected as the object of study on the motion reliability analysis of flexible mechanism, to validate the proposed ENLM. The simplified model of TFRM is shown in Fig. 4.

In the TFRM, the members of the manipulator are a homogeneous Euler beam. Mass loads enforce at the joint between component 1 (denoted by member-1) and component 2 (denoted by member-2). The end of arm is assumed to be concentrated masses without the consideration of rotational inertia and damping of rotor motor. For the two manipulators, the lengths are l_1 and l_2, the masses are m_1 and m_2, and the driving torques are $\boldsymbol{\tau}_1(t)$ and $\boldsymbol{\tau}_2(t)$, respectively. To simulate the motion conditions of the two manipulators (members), the local coordinate systems x_1-o-y_1 and x_2-o_1-y_2) are built for member-1 and member-2 in Fig. 4, respectively. y_1 and y_2 stand for the elastic deformations of member-1 and member-2, respectively. t is the movement time. The movements of two moving local coordinates are described by using the azimuth angles $\boldsymbol{\theta}_1(t)$ and $\boldsymbol{\theta}_2(t)$.

Based on the integrated mode method [26], the elastic deformations of the two members in the corresponding local coordinate systems are analyzed. The shape functions ϕ_1 for number-1 and ϕ_2 for number-2 is expressed by

$$\begin{cases} \phi_1(x) = \sin\left(\frac{\pi x}{l}\right) \\ \phi_2(x) = \sin\left(\frac{2\pi x}{l}\right) \end{cases} \tag{12}$$

The elastic deformations of the two components vary with time. Thus, the elastic deformations $y_1(t, x_1)$ and $y_2(t, x_2)$ for member-1 and member-2 on y direction are computed respectively by

$$\begin{cases} y_1(t, x_1) = \sum_{i=1}^{n} g_i(t)\phi_i(x_1) \\ y_2(t, x_2) = \sum_{i=1}^{n} u_i(t)\phi_i(x_2) \end{cases} \tag{13}$$

The generalized coordinate $\boldsymbol{q}(t)$ is gained as

$$\begin{aligned} \boldsymbol{q}(\mathrm{t}) &= [q_1, q_2, q_3, q_4, q_5, q_6]^T \\ &= [\theta_1(t), g_1(t), g_2(t), \theta_2(t), u_1(t), u_2(t)]^T \end{aligned} \tag{14}$$

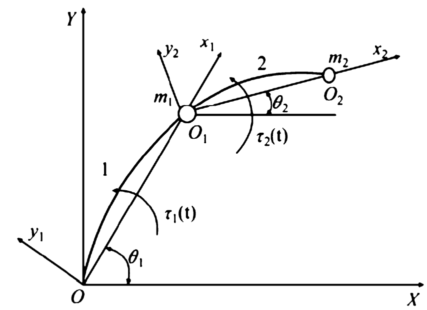


Fig. 4. Schematic diagram of two-link flexible robot manipulator.

Table 1
Basic parameters of member-1 and member-2.

Parameters	Mass M, kg	Length L, m	Drive torque τ, N·m
Member-1	5.5	0.75	$215\sin^3(2\pi t)-62$
Member-2	7.5	0.75	$75\sin^3(2\pi t)+15$

where $g_i(t)$ $(i = 1, 2)$ is the ith elastic coordinate of member-1; $u_i(t)$ the ith elastic coordinate of member-2.

According to the Lagrange equations, the kinetic equation of the TFRM is expressed by

$$\boldsymbol{Q}_k = \boldsymbol{M}\ddot{\boldsymbol{q}} + \dot{\boldsymbol{M}}\dot{\boldsymbol{q}} - \frac{\partial}{\partial \boldsymbol{q}}\left(\frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{M}\dot{\boldsymbol{q}}\right) + \boldsymbol{K}\boldsymbol{q} + \frac{\partial \boldsymbol{U}_g}{\partial \boldsymbol{q}} \tag{15}$$

where $\boldsymbol{U}_{\boldsymbol{g}}$ is the gravitational potential energy of the system; $\boldsymbol{M}$ denotes the mass matrix; $\boldsymbol{K}$ indicates the stiffness matrix; $\boldsymbol{Q}_k(t)$ expresses the total force corresponding to the moment of rotation calculated by the virtual work method [46,47].

The failure modes of the TFRM involve deformation failure and strength failure. Generally, the dynamic strength of TFRM is not easy to fail. Therefore, the main failure mode of the TFRM is deformation failure [26]. In this paper, we focus on TFRM deformation to investigate the motion reliability analysis of flexible mechanism.

4.2. Variables selection

In respect of the motion of the TFRM, mass M, length L and driving torque $\boldsymbol{\tau}$ are considered as the basic parameters as shown in Table 1. The density ρ, elastic modulus E and section size h and b for the two of manipulators TFRM are regarded as the random parameters of the motion reliability analysis of TFRM which are shown in Table 2, in which the variables are assumed to follow normal distributions and to be independent mutually.

4.3. ENLM modeling

In respect of the distribution feature of input random variables in Table 2 and the LHS technology [44], 150 groups of samples are extracted for input variables. Then, based on the collected samples and the kinetic equations of TFRM in Eq. (15), the output responses (maximum deformations) of member-1 and member-2 are calculated as the samples together with the acquired input samples. From the pool of samples, 120 groups of samples are regarded as training samples for the ENLM modeling, and the remain 30 groups of samples are token as the test samples to evaluate the built ENLM model. It should be noted that all the samples are normalized before used. To determine the number of neurons (i.e., nodes), we mark network hidden layer neuron by $k_i = 2 \sim 9$ $(i = 1, 2)$, for the two members of mechanism system. In this case, the number of hidden layers n is computed by

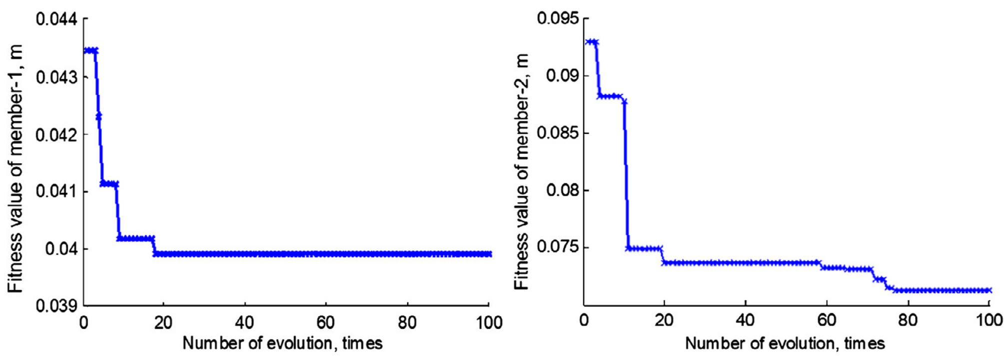
$$n = \sqrt{n_i + n_o} + a \tag{16}$$

Table 2
Random parameters of member-1 and member-2.

Variables	Density ρ, kg·m^{-3}	Modulus of elasticity E, Pa	Member-1		Member-2	
			h_1, m	b_1, m	h_2, m	b_2, m
Mean	2067	4.0875×10^9	0.06	0.015	0.04	0.01
Standard deviation	10	2.0438×10^8	0.04	0.01	0.0267	0.0067

Table 3
Network training error with different hidden neuron number.

Neuron number	2	3	4	5	6	7	8
Network-1 error	0.12	0.10	0.13	0.13	0.12	0.20	0.22
Network-2 error	0.17	0.16	0.16	0.17	0.19	0.18	0.20



(a) Member-1 (b) Member-2

Fig. 5. Optimal fitness value curves.

in which n_i is the number of input nodes; n_o denotes the number of output nodes; a expresses the arbitrary constant [19,20]. For the different number of neurons, the network training errors are computed as listed in Table 3.

In line with the comparison of the network training error in Table 3, the network training error is minimal as the number of hidden layer nodes ($k_1 = k_2 = 3$) is selected for the ENLM modeling. Hence, in light of the number of input variables (4), the number of hidden layer nodes (3) and the number of output response (1, only consider one failure mode (deformation)), we select the 4-3-1 three-layer network structure as the general ENLM model. In this model, the transfer functions from input layer to hidden layer and hidden layer to output layer choose 'tansig' and 'purelin', respectively. When particle dimension $v = 19$ and particle number $N = 40$, by 100 iterations the optimal adaptive value curves in the two transfer functions ('tansig' and 'purelin') are shown in Fig. 5.

As illustrated in Fig. 5, with the increase of evolution the adaptive values of member-1 and member-2 reach to the stability with 0.0399 m and 0.0712 m, respectively, after 20 evolutions and 80 evolutions (or iterations). The optimal adaptive values are 0.0399 m and 0.0712 m for member-1 and member-2, respectively, in respect of the two transfer functions ('tansig' and 'purelin'). Respecting the initial weights and threshold values given arbitrarily and 120 groups of training samples, the ENLM model is trained to compute the hyperparameters $\boldsymbol{\theta}^1$ and $\boldsymbol{\theta}^2$ (i.e., weight and threshold levels) for the two members, which are shown in Eq. (17) and Eq. (18).

Member-1:

$$\boldsymbol{\theta}^1 = \begin{cases} \boldsymbol{LW}^1_{1,1} = \begin{bmatrix} 0.3189 & 1.3753 & \cdots & 0.0442 & -1.5461 \\ 0.1539 & 0.5896 & \cdots & 0.1539 & 0.6162 \\ 1.7788 & 1.0573 & \cdots & 3.6510 & 0.2696 \\ 2.1675 & -0.4763 & \cdots & -3.2551 & 1.5656 \end{bmatrix}_{1\times120} \\ \boldsymbol{LW}^1_{2,1} = [-1.31590.5464 \cdots 1.0812 - 0.8796]_{1\times120} \\ \boldsymbol{b}^1 = [-0.36970.8757 \cdots 0.01170.5153]_{1\times120} \end{cases} \tag{17}$$

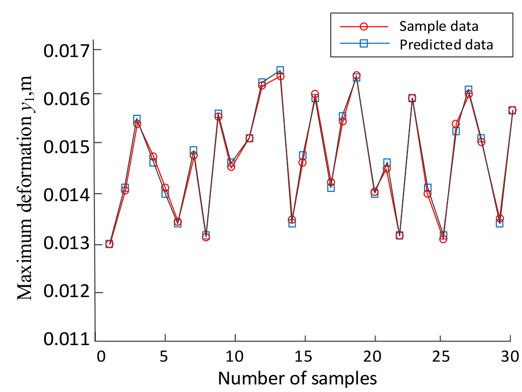


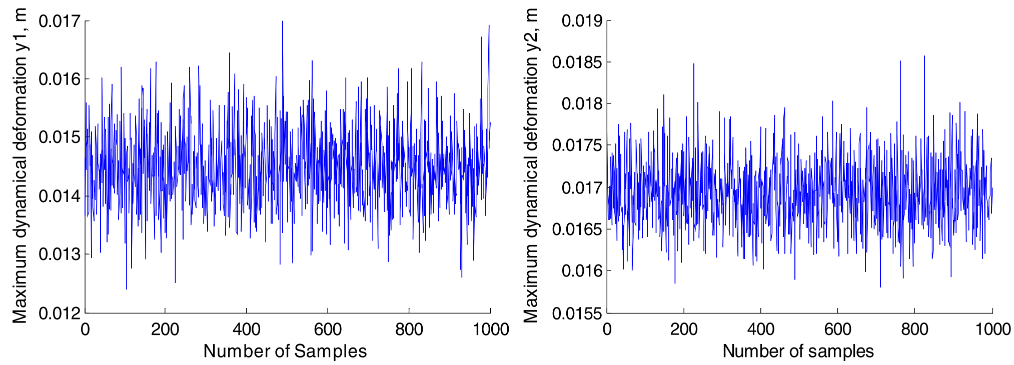
Fig. 6. Test results of the built ENLM model with 30 samples.

Member-2:

$$\boldsymbol{\theta}^2 = \begin{cases} \boldsymbol{LW}^2_{1,1} = \begin{bmatrix} -2.2581 & -2.2921 & \cdots & 0.8942 & -2.9423 \\ 0.1191 & 0.3811 & \cdots & 0.9815 & -1.5894 \\ -0.3882 & 2.3953 & \cdots & 0.9412 & 0.2033 \\ -0.0081 & -1.6267 & \cdots & 0.6289 & 0.0782 \end{bmatrix}_{1\times120} \\ \boldsymbol{LW}^2_{2,1} = [2.94631.9262 \cdots - 0.3262 - 0.5941]_{1\times120} \\ \boldsymbol{b}^2 = [0.9115 - 0.4468 \cdots - 0.2628 - 0.3359]_{1\times120} \end{cases} \tag{18}$$

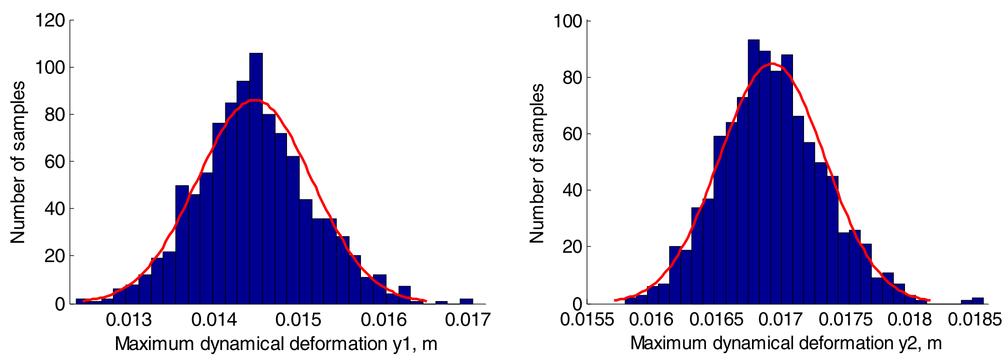
By inputting the values of the parameters into Eq. (9), the ENLM model was built. To support the validity of the established ENLM model, the 30 groups of test samples are used to test the established ENLM model of member-1. The prediction results are displayed in Fig. 6.

As seen in Fig. 6, in respect the 30 test samples, the predicted data with the established ENLM model are almost consistent with



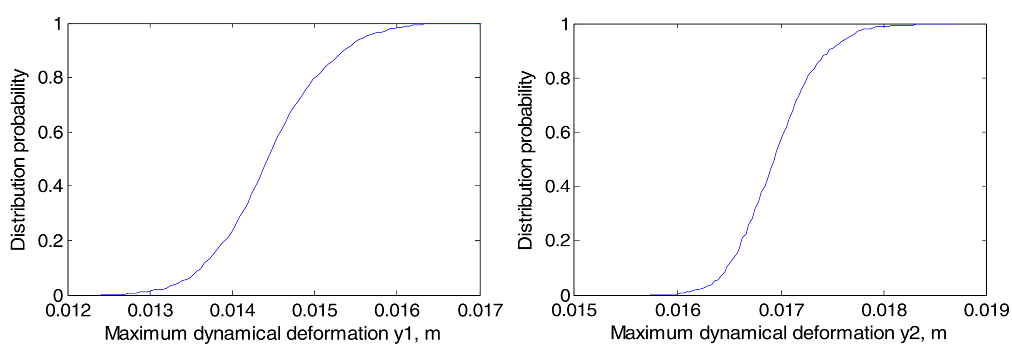
(a) Member-1 (b) Member-2

Fig. 7. Maximum deformation of midpoint of two members.



(a) Member-1 (b) Member-2

Fig. 8. Maximum deformation distribution of two members.



(a) Member-1 (b) Member-2

Fig. 9. Deformation cumulative curves.

the true sample data, because only there are very small errors for each test sample. The small error indicates the high prediction precision of the developed ENLM as well as the validity and feasibility of ENLM based on the samples of flexible mechanism (deformation). Therefore, the built ENLM model can be applied to perform the motion reliability analysis of TFRM in coming section.

4.4. Motion reliability analysis of flexible mechanism

By using the MC method and LHS technology, the built ENLM model is simulated 1 000 times, in respect of the distributions of random variable s in Table 2. The deformation curve, deformation distribution and deformation cumulative function of two members are gained in Figs. 7–9. Assuming that the allowable deformation is 1.8×10^{-2} m, the results of reliability analysis are listed in Table 4.

As shown in Fig. 7–Fig. 9, the maximum dynamical deformation values of two members are evenly distributed around the mean values, and approximately obey a normal distribution with the mean values 1.45×10^{-2} m and 1.69×10^{-2} m and the standard deviations 6.77×10^{-4} m and 4.08×10^{-4} m, respectively.

As revealed in Table 4, the reliability degrees for member-1 and member-2 are 1 and 0.951, respectively, as the allowable deformation is 1.8×10^{-2} m. The TFRM is a continuous system. The system strength reliability is equal to the product of the reliability degrees of members [1]. Thus, the reliability degree of TFRM system is $P_y = P_{y1} \cdot P_{y2}$=0.951. Through 1 000 simulations on the BP-ANN models for two members, the computational time of two members is 3.102 s and 3.127 s respectively, which involve both modeling time and simulating time of ENLM model. It is illustrated that it is acceptable for the reliability degrees and simulating time for member-1, member-2 and TFRM system, which indicate that the developed ENLM is efficient and feasible in the motion reliability evaluation of flexible mechanism.

Table 4
Results of TFRM deformation reliability evaluation.

Components	Failure number	Reliability	Mean, $\times 10^{-3}$ m	Standard deviation, $\times 10^{-3}$ m	Distribution	Computational time, s
Member-1	0	1	14.5	0.677	Normal	0.126
Member-2	49	0.951	16.9	0.408	Normal	0.158

Table 5
Computing time of TFRM reliability analyses.

Methods	Number of simulations, times			
	10^2	10^3	10^4	10^5
MC method	29.4 s	298 s	2 991 s	30 057 s
QP-RSM	1.67 s	4.62 s	12.93 s	325.91 s
QP-ERSM	0.36 s	0.59 s	1.68 s	125.28 s
GA-GRNN	0.27 s	0.45 s	1.13 s	97.19 s
ENLM	0.14 s	0.28 s	0.49 s	36.08 s

4.5. Method validation

To further support the proposed ENLM, the motion reliability analyses of TFRM are conducted with respect to MC method and four surrogate models under different simulations (i.e., 10^2 times, 10^3 times, 10^4 times and 10^5 times) based on the same computational conditions and computer environment. The four surrogate models are quadratic polynomial (QP)-RSM, QP-ERSM, GA-GRNN and ENLM. The QP-RSM and QP-ERSM is based on quadratic polynomials and solved by least square method. The model of the GA-GRNN is structured by the GA to find the optimal model parameters, and the MPGA is adopted to search for the optimal parameters of ENLM model. All analyses are performed by one Intel Pentium 4 desktop computers with 2.13 GHz CPU and 8 GB RAM. Through these analyses, the compared results in computational time and reliability degree are listed in Table 5 and Table 6. In Table 6, the precision (pr) of TFRM reliability analysis for N simulations is defined as

$$pr(N) = 1 - \frac{|R_{\text{mod el}}(N) - R_{\text{MC}}(N)|}{R_{\text{MC}}(N)} \tag{19}$$

where R_{model} is the reliability degree gained by surrogate model methods like QP-RSM, QP-ERSM, GA-GRNN and ENLM; R_{MC} is the reliability degree gained by MC method; N is the number of simulations.

As revealed in Table 5, (i) the surrogate model methods (i.e., QP-RSM, QP-ERSM, GA-GRNN and ENLM) took much less computational time than the MC method. For example, under the 10^4 simulations, the MC method costed 2991 s while other four surrogate models (QP-RSM, QP-ERSM, GA-GRNN and ENLM) only spent 12.93 s, 1.68 s, 1.13 s and 0.49 s, respectively. The reason is that the solution of the differential equations in Eq. (15) based on the MC method needs numinous iterations, while the surrogate models (ERSM and ENLM) did not require in probabilistic simulations. In other words, for each simulation, the surrogate model is directly simulated without iterations, while the MC method needs a large number of iterations in the solving process of the differential equations; (ii) Relative to QP-RSM, the QP-ERSM took shorter computational time under the same simulation number. For instance, under 10 000 simulations, the QP-RSM spent 12.93 s while the QP-ERSM costed 1.68 s. This is because ERSM only considers one extremum value to establish one surrogate model during time domain as the ERSM is modeled, while RSM needs to build many models at different time points in the time domain range; (iii) comparing to the QP-ERSM an GA-GRNN, the developed ENLM needs less time to complete the same simulations. This is because the MPGA adopted many GA to synchronously find the optimal parameters in the ENLM model due to fast optimizing operators, relative to the quadratic polynomial and least square method in searching for the optimal parameters of the ERSM and the GA for finding the optimal GA-GRNN model parameters; (iv) with the increase of simulations, the computing time for all methods rise. What's more, the high efficiency of the surrogate model methods (i.e., QP-RSM, QP-ERSM, GA-GRNN and ENLM) becomes more obvious with the rise of simulation number. For instance, at the same 10^4 simulation, the time cost of the ENLM (~0.49 s) is about 1/4063 that of MC method (~2991 s), 1/26 that of QP-RSM (~12.93 s), 2/7 that of QP-ERSM (~1.68 s) and 2/5 that of GA-GRNN (~1.13 s), while at 10^2 simulations the time cost of ENLM only 1/210 that of MC method, 1/11 that of QP-RSM, 2/5 that of QP-ERSM and 1/2 that of GA-GRNN. Therefore, the proposed ENLM with MPGA has high computational efficiency and is promising to highly-efficiently implement the motion reliability analysis of flexible mechanism.

In Table 6, it is observed that, (i) the QP-ERSM is more accurate than QP-RSM. This is because the QP-ERSM simplified the response process as one globally extremum value in ERSM modeling, which avoids the local optimization problem, which address the transient problem in dynamic reliability analysis and then ensure the effectiveness of samples used to surrogate modeling; (ii) compared to the QP-based surrogate models (i.e., QP-RSM and QP-ERSM), the GA-GRNN and ENLM hold higher accuracy. The reason is that the GRNN model has outstanding advantage (strong mapping ability) in describing the nonlinear problem of the true model and this strength guarantees the precision of the built GRNN model; (iii) the presented ENLM in this study reaches to the precision of over 0.999. In other words, the precision of the ENLM in the motion reliability analysis of TFRM is almost consistent with MC method in the calculation accuracy and is higher than other surrogate models (QP-RSM, QP-ERSM, GA-GRNN) in simulating accuracy. It is noted that the MC method is to directly simulate the differential (motion) equations of TFRM. It is revealed that in the proposed ENLM and the motion reliability evaluation of flexible mechanism, the GRNN model efficiently handles the high-nonlinear problem, the extremum thought of ERSM contributes to process the transient characteristics and local optimization problems, and the adopted MPGA accurately finds the parameters of ENLM model. In other words, the three techniques (ERSM thought, GRNN model and MPGA) together determine the modeling precision of ENLM, which ensure the reasonability of TFRM motion reliability analysis.

In conclusion, the proposed ENLM holds high modeling precision and high simulation efficiency in the motion reliability analysis of complex mechanism, by adopting the GRNN to perfectly handle high nonlinearity, and applying the ERSM to address the transients, and utilizing the MPGA to find the optimal model parameters.

5. Conclusions and future works

The objective of this paper is to develop an enhanced network learning method (ENLM) by integrating the strengths of generalized regression neural network (GRNN), extremum response surface method (ERSM) and multi-population genetic algorithm (MPGA), to improve the modeling precision and simulation efficiency of the motion reliability analysis of flexible mechanism. The reliability analysis of two-link flexible robot manipulator (TFRM) is investigated. Some conclusions are summarized as follows:

Table 6
Precision of TFRM reliability analyses.

Sample number	Reliability degree					*Pr*, %			
	MC method	QP-RSM	QP-ERSM	GA-GRNN	ENLM	QP-RSM	QP-ERSM	GA-GRNN	ENLM
10^2	0.97	0.95	0.98	0.97	0.97	98	99	100	99
10^3	0.952	0.936	0.961	0.953	0.951	98.3	99.1	99.9	99.9
10^4	0.9865	0.9496	0.9538	0.9778	0.9866	96.26	96.70	99.12	99.99
10^5	0.98513	0.95123	0.95687	0.98497	0.98611	96.559	97.131	99.984	99.901

(1) The total reliability degree of TFRM is 0.951 and the reliability degrees of member-1 and member-2 are 1 and 0.951, respectively. Meanwhile, the maximum deformations of member-1 and member-2 follow the normal distribution. It is illustrated that the use of ERSM can reasonably process the transient problem to implement the motion reliability analysis of flexible mechanism.
(2) The presented ENLM can effectively process the transient problem in the motion reliability analysis of flexible mechanism, by the ERSM simplifying the process of output response as a specific global value. The ENLM holds the potential of improve the modeling precision and simulating speed.
(3) The developed ENLM can approximate to the nonlinear problem and make the built ENLM model accurately approximate to the true problem, by finding the optimal model parameters with the assist of MPGA.
(4) The advantages of ENLM are more obvious with the increase of simulation number, which demonstrates that the ENLM model with MPGA has higher computational efficiency and precision and better robustness.
(5) The effort of this paper provides a promising way for the motion reliability analysis and optimal design of flexible mechanism with high modeling precision and simulation efficiency.

In light of this study and the raised questions, two issues should be resolved in the coming work. One is that the developed ENLM should be extended to engineering fields such as satellite, spacecraft, robotics, aeroengine, and so forth, in respect of complex loads, and the other is that the ENLM model should be refined in future by more reasonable analysis techniques and optimization algorithms such as deep learning algorithms.

Declaration of competing interest

The authors declare that there is no conflict of interests regarding the publication of this article.

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